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COMPETITIVE LOCATION MODELS: A FRAMEWORK AND BIBLIOGRAPHY

by

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ABSTRACT

This paper presents a taxonomy for competitive location models, based on the following components: the space, the number of players, the pricing policy, the equilibrium concept of the game and the behavior of the customers. The components are briefly discussed and based on these, an annotated bibliography comprising more than 100 selected titles follows.

Key words : competitive location models.

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RÉSUMÉ

Cet article présente une taxonomie pour les modèles de localisation compétitive et basée sur les composantes suivantes: l'espace, le nombre de joueurs, la politique du prix, le concept d'équilibre du jeux et le comportement des clients. L'article inclut une brève discussion de ces composantes ainsi qu'une bibliographie annotée, basée sur ces composantes et comprenant plus de 100 titres sélectionnés.

Mots-clés : modèles de localisation compétitive.

1. INTRODUCTION

The study of competitive location models is based on the seminal paper of Hotelling[14]. In his work, Hotelling studies the strategies of two competitors with respect to price and location under a number of idealistic assumptions. This work initially stimulated a number of responses and extensions (see, e.g. Lerner and Singer[16] or Smithies[29]). Since the 1970s, a steady flow of papers in the field has been published. This has dramatically increased in the 1980s. Today, the field is firmly embedded in spatial economics and industrial organization. The reader is referred to Ponsard[23] for a historical overview of the theme as well as to Graitson[10], Gabszewicz and Thisse[8,9] and Eiselt and Laporte[5] for surveys of the topic.

However, economists are not alone in their interest in this discipline. For a long time, geographers have studied locational patterns emerging from some kind of interaction or competition between facilities. Well-known early contributions are those by Christaller[3] and Losch[17]. Clearly, the points of view of the two groups are different: whereas economists tend to be more concerned with prices, profits, and welfare considerations, geographers are usually more interested in locational configurations and market areas. Today, mathematicians, operations researchers, political scientists and many others publish in the field of location theory. This has become one of the truly interdisciplinary fields of study.

From a methodological point of view, the main distinctive feature of competitive location models in contrast to standard location models is the use of arguments from game theory. In essence, game theory is the mathematical discipline concerned with the behavior of decision makers (players) whose decisions affect each other; see, for example, Moulin[20] for a good introduction to game theory as well as Friedman[7] for a more advanced textbook. According to a well-established tradition, games are divided into two classes: cooperative and noncooperative games. A game is said to be cooperative when players can make binding agreements; it is called noncooperative when they cannot. In this work, we will be concerned with applications of noncooperative games only. It should be noted however that some recent papers have applied cooperative game theory to location problems. Recent references are Granot[11], Hamilton, MacLeod and Thisse[13], and Tamir[30].

The increasing interest in competitive location modeling is not only a result of vastly increased computational power and algorithmic knowledge (e.g. without either, a study such as that by Okabe and Suzuki[22] would be unthinkable) but also that of an increased number of known applications. With few exceptions, early studies considered the location of physical objects such as industrial plants, warehouses, etc. But even Hotelling himself had suggested applying his model to a scenario involving the location of objects in an abstract space. In particular, he proposed to define an "issue space" by identifying a dimension with each relevant political issue and then locating political candidates in that space. This model was elaborated upon by Downs[4] and Black[1] in their classical

treatises. Good modern surveys of the topic are those by Enelow and Hinich[6] and Mueller[21]. Today, similar models are used by economists and marketing scientists to locate products in attribute spaces, personnel planners locating (potential) employees in ability spaces, and many others; for a recent survey and references in marketing, see Schmalensee and Thisse[26].

It is obvious that a multitude of factors influence the choice of a location, even in the absence of competition. These factors clearly depend on the specific application. In traditional plant location models, the distribution of demand, the availability of skilled labor, transportation costs, prices, availability of appropriate building lots, and many other considerations have to be observed. The situation becomes even more complex under competition. For example, are facilities located simultaneously or sequentially? Are only locations or also prices decision variables? What are the objectives of the decision makers? How do customers behave, do they just patronize the closest facility or are other factors considered as well?

2. A TAXONOMY

In this section we will introduce what we consider to be the major components of competitive location models. Based on this discussion a taxonomy is suggested. This taxonomy is a quintuple I/II/III/IV/V, where

I identifies the space;

II specifies the number of players;

III indicates the pricing policy;

IV specifies the equilibrium concept of the game and

V states the behavior of the customers.

These components are briefly introduced and discussed in subsequent paragraphs.

A number of different spaces have been employed in the literature. The simplest space is a (one-dimensional) line segment usually referred to as a linear market. Hotelling's original model uses this space, here symbolized by "|L|". In order to avoid boundary effects, Chamberlin[2] suggested the use of a circular market "C". Another possibility is to employ an unbounded line "L". Whereas these models are restricted to only one dimension, the most general of the metric spaces employed in competitive analyses is the m-dimensional real space "R^m". Note that multidimensional spaces are especially relevant for the choice of a political platform and for the selection of a product. In most instances, this space will be bounded (e.g., by a rectangle or a circle) in which case it is denoted by $|R^m|$. A different type of space is provided by networks. As usual, a network is defined as a set of vertices and a binary relation defined on the vertices, the elements of which are called edges. The designation network "N" encompasses all directed, undirected, and mixed graphs and networks. One special case is that of a tree "T", defined as an undirected connected graph without cycles. Of all spaces introduced above, only in R^m and $|R^m|$ is the issue of distance relevant. Most competitive models use the Euclidean (ℓ_2) distance function whereas studies in optimal (noncompetitive) location also

employ the Manhattan (ℓ_1), the Chebyshev (ℓ_∞) as well as general ℓ_p distance functions. Due to the restricted relevance of distance functions in the models discussed here, they are not considered in our taxonomy.

The second component in our taxonomy denotes the number of competitors or players. Each player will locate any prespecified number of facilities in a given space. Many models study the case of two players only. Specific results may also arise with three players. More generally, the number of players (here denoted by "n") may be given but unspecified. An important area of study in economic theory discusses models where players can locate facilities on a market as long as they can operate profitably, so-called "markets with free entry". In such models the number of players and facilities is not given a priori and we use a "?" in the classification scheme.

The third component in our taxonomy concerns the pricing policy used by the players. As indicated above, we distinguish between models where location and price are decision variables as opposed to models in which prices are fixed and location is the only decision variable. Obviously, the latter class of models is simpler. The case of fixed or null prices is shown by "empty set" in the classification. Fixed or nonexistent prices occur in many areas of study. In the case of competing public facilities, players (e.g., jurisdictions) usually maximize a welfare function specific to a group of users without resorting to price (free access). Considering a model involving political parties, the vote shares of the parties are the major concern. Similarly, public authorities or cartels may regulate prices. In

such cases, the usual objective of profit maximization reduces to sales maximization if constant marginal costs can be assumed.

If price is also a decision variable, we have to distinguish between a number of different spatial pricing policies. The first is mill (or f.o.b.) pricing (here denoted by "M") where the good is sold at the facility for the same price to all customers who then have to pay the full transportation costs. The second possibility is delivered pricing where the facility provides for (and pays) the transportation costs. Delivered pricing policies come essentially in two versions. Either the firm delivers the good to all customers for the same price - uniform delivered pricing "U" - or the firm charges for the good depending on where they are delivered to - a system called (perfect) spatial discriminatory pricing, here shown as "D". As an example, some mail order firms use uniform delivered pricing (such as L.L. Bean in the U.S.) whereas some steel producing companies use spatial price discrimination in the E.E.C. Note that uniform prices actually subsidize customers who are more distant from the facility and that there are some legal restrictions regulating price discrimination.

The fourth component in our taxonomy deals with the equilibrium concept of the game. In essence, we distinguish between simultaneous and sequential location of facilities. Let us first examine the simultaneous choice of locations in the context of a noncooperative game. A natural solution of the game would be one where none of the players has any incentive to unilaterally change his decision variables. The corresponding solution concept is called a (pure strategy) Cournot - Nash equilibrium,

here abbreviated by "C". Given fixed or nonexistent prices, a locational Cournot-Nash equilibrium is a configuration of locations where none of the players could improve his own objective by relocating unilaterally. As a simple example, consider a duopoly on a linear market with fixed prices, constant costs and uniform demand. If both players locate their respective facilities arbitrarily close to each other at the center of the market, a Cournot-Nash equilibrium has been reached. Note that such an equilibrium does not always exist; one such instance is the 3-facility model on a linear market as described in the early paper by Lerner and Singer[16].

Two different approaches are usually considered when prices are also variable. Most models in this context follow Hotelling's formulation and use a refinement of the Cournot-Nash equilibrium. More precisely, it is supposed that players choose location and price, one at a time in a two-stage process, with the aim of maximizing their own profits. The division into two stages is motivated by the fact that the choice about location is usually prior to the decision on price. In the first stage, players simultaneously choose their location. Given any outcome of the first stage, players then simultaneously choose their price in the second stage. The second stage can be modeled as a noncooperative game in which strategies are prices and payoffs are profits. This game is solved at the Cournot-Nash price equilibrium, assuming that such an equilibrium exists. The resulting second-stage profits then depend only upon the location decisions made by the players in the first stage. Accordingly, these profits can be used as payoffs in the first-stage game in which strategies are locations. This game, in turn, is solved at the Cournot-Nash location equilibrium. The

corresponding two-stage solution concept is called a subgame perfect Nash equilibrium denoted by "P". It captures the idea that, when players select their location, they all anticipate the consequences of their choice on price competition. However, this solution concept is not used by all the authors. A few of them suppose a simultaneous choice of price and location, for which we still use "C".

An alternate approach is that of sequential moves which allows a player to choose a location anticipating subsequent decisions by future competitors. Specifically, at each stage of the entry process, the corresponding player takes as given the locations of facilities established at earlier stages, but can treat the locations of facilities established at later stages as conditional upon his own choice. The location chosen by a player is then obtained by backward induction, from the optimal location selected by the ultimate entrant to the player himself. Such a solution is also referred to as a (von) Stackelberg equilibrium; thus we abbreviate it by "V". In this family of models, it is usually assumed that facilities make their locational decisions once-and-for-all. This hypothesis is made to express the fact that changing the location of a plant and redesigning an existing product is often prohibitively costly. One major reference in this domain is Prescott and Visscher[25]. The main disadvantage of this approach is that models become very messy as the number of competitors increases.

Most operations research studies investigating location models under competition do not readily fall into the categories outlined above. Following Slater[28] and Hakimi[12], most studies consider the loction of

new facilities given that facilities of competing players already exist. Conditional locations of this type are called centroids and medianoids. As no equilibria are sought, we use a dash "-" for such analyses.

The last component in the taxonomy concerns the behavior of customers. In general, we have to distinguish between deterministic and probabilistic models. In the former case, customers are attracted to a facility according to some given function and it is assumed that they will always patronize the single facility they are most attracted to. In the case of probabilistic models, a similar attraction function is given that specifies probabilities that a customer patronizes a facility. The major difference between the two classes of models emerges when assigning customers to facilities. In case of a deterministic model, each facility can determine the set of customers more attracted to it than to any other facility. This set is then called the market (or catchment) area of the facility under consideration. The collection of such market areas in these so-called market area models is a tessellation of space that in some special cases coincides with a Voronoi diagram. Some standard references are Shamos and Hoey[27] or Preparata and Shamos[24]. On the other hand, the concept of market areas in probabilistic models becomes meaningless as each facility, even the least attractive one, has some chance of being patronized by a customer.

As far as our taxonomy is concerned we will use the following conventions. Consider the deterministic models first. The most popular assumption is that customers choose the facility on the basis of distance, (here abbreviated by "D") more specifically the one that is closest to them,

where the term "close" again assumes some underlying metric. Even though the logic behind this assumption appears compelling, there are a number of scenarios where customers do not behave accordingly. For instance, customers may choose the firm with the lowest full price (mill price plus transportation cost) which is not necessarily the closest facility. Models of this type are rather important in describing shopping behavior. Similarly, in the case of nonexistent prices such as the location of social services (e.g., child care facilities), customers may patronize the facility that is more accessible (e.g., on their way to work) rather than the closest one. Whatever factors other than distance influence customer behavior in a deterministic model, we use the term "U" for utility (or attraction) function. One example of this type is the work by Kohlberg[15] who assumes that customers patronize facilities on the basis of travel time (a function of distance) plus waiting time (a function of market share).

In a probabilistic model, a customer is attracted to a facility according to some function, e.g inversely proportional to the customer-facility distance. The probability that a customer patronizes a facility could then be defined as the magnitude of attraction to the facility in relation to the sum of all attractions. The most popular model in this class is the logit model; see Luce[18]. Most probabilistic models used result from the maximization of a random utility function and are denoted by R; see for instance McFadden[19].

In summary, the components in the taxonomy are:

I: Space: |L|: linear segment

L: line

C: circle

$|R^m|$: bounded subset of m-dimensional real space

R^m: m-dimensional real space

N: network

T: tree

II: Number of Players: 2, 3, ...: specified number

n: any arbitrary fixed number

?: markets with free entry

III: Pricing Policy: Variable prices: M: Mill pricing

U: Uniform delivered pricing

D: Perfect spatial discriminatory pricing

Ø: Nonexistent or fixed prices

IV: Equilibrium Concept of the Game: C: Cournot-Nash equilibrium

P: Subgame perfect Nash equilibrium

V: (von) Stackelberg equilibrium

-: The study does not consider

equilibria

V: Customer Behavior: D: Minimization of distance

U: Maximization of a deterministic utility

R: Maximization of random utility

The bibliography in the next section lists and codes some of the most relevant references in the field according to this taxonomy.

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